Supply Chain Coordination Under Financial Constraints and Yield Uncertainty

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Abstract: A coordination problem for a supply chain with capital constraints and yield uncertainty is considered in this paper. In order to improve the supply chain, a buyback and risk sharing (BBRS) mechanism is proposed, in which the distributor shares the supplier's yield uncertainty risk by purchasing the overproduced products or waiving the shortage penalty, and the supplier shares the distributor's demand uncertainty risk by buying back the unsold products. The results indicate that, the profits and the strategies under the BBRS are the same with those under the centralized case. In addition, the proposed BBRS mechanism has a built-in mechanism to allocate the spillover profit between the supplier and the distributor. The results also show that the BBRS can increase the production quantity. Finally, we derived the bankruptcy probabilities for both the supplier and the distributor, and the probabilities depend on the initial capitals.

Keywords: Supply Chain Finance; Capital Constraint; Yield Uncertainty; Buyback; Risk Sharing

1 Introduction.

Capital constraint is very common in supply chains. Due to capital constraints, suppliers often have capital deficits during the production period, and distributors often have capital deficits during the sales period. Usually, those capital deficits are financed by bank loans. There have been lots of literature on this topic (see the literature review in Subsection 2.1).

In addition, the supplier often faces a productivity yield uncertainty. For example, the production processes in semiconductor and electronics industries are highly uncertain and it is common to expect a yield of 50% in the small and medium LCD manufacturing industry (Chen and Yang, 2014). Another very common yield uncertainty occurs in agricultural production systems which include productions of olive oil, orange juice, timber or hybrid corn seed and other agricultural products. For example, the actual yield for olive oil production in Turkey can be as low as 30% or 40% (Kazaz 2004).

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The yield uncertainty not only has negative effects on suppliers' decisions, but also brings some bankruptcy risks to banks who issue loans to the suppliers. The traditional bank loan financing solution may still work, but the bank needs to consider the supplier's bankruptcy risk, that is, the supplier may not be able to pay back the loan when the yield is not as strong as expected. When this bankruptcy risk is taken into consideration, the bank tends to charge a higher interest rate. Despite that, in practice, applying for a bank loan is still the most popular solution to capital deficits. Therefore, to be realistic, we still use bank loans as the solution to finance the capital deficit. The main focus of this paper is to investigate how to improve the supply chain performance by virtue of some coordination contract.

Although there have been many research works on the financing and coordination strategies for supply chains with capital deficits, to our best knowledge, not so many have ever considered productivity yield uncertainty. In this paper, we fill the gap by considering a supply chain with demand uncertainty, productivity yield uncertainty and capital constraints. We first derive the optimal operation strategies for the supplier and the distributor under the decentralized case and the centralized case. Then, we propose a mechanism of buyback and risk sharing (BBRS), under which the supplier buys back the unsold products at the end of the sales season to share the risk of demand uncertainty, and the distributor buys the overproduced products or exempt the supplier from shortage penalty to share the risk of yield uncertainty. As we can see, the proposed BBRS has a risk sharing feature. This is the main difference between the BBRS and traditional buyback contracts. Further analysis indicates that, under the BBRS, the profits and the strategies of the supply chain are the same with those under the centralized case. Therefore, the BBRS can solve the capital constraint issue for the supply chain with financial constraints and yield uncertainty efficiently.

The above proposed BBRS coordination contracts can be well accepted in practice. For example, it is very common in China that the dealers of agricultural products purchase all of the agricultural products from farmers, including the portion of the products that exceeds the order. Due to the market demand uncertainty, the ordered agricultural products may not be sold out completely. In this case, the farmers usually buy back some of the unsold products with a lower price to share the risk of demand uncertainty. Another very common example is the grocery supermarket industry. In China, a supermarket (such as Walmart, Carrefour, etc.) usually signs a contract with a fruit/vegetable supplier to order as much as possible at the beginning. While, at the end of the sales season, the supplier buys back the unsold products with a pre-agreed price. This model is becoming more and more popular for some online supermarkets in China, such as Jingdong, Hema, etc.

The reason that the proposed BBRS contract works is that, for the capital constrained supply chain with yield uncertainty, it can be coordinated if the risk of productivity yield uncertainty and the risk of market demand uncertainty are both shared between the distributor and the supplier. Some sensitivity analysis with respect to the interest rates, and initial capitals are performed in this paper. Numerical results are presented, followed by some managerial insights and more detailed discussions. Some interesting results are found and they may give the direction to further improve the supply chain. For example, the sensitivity analysis with respected to interest rates suggests that government subsidy to financing costs may improve the production and ordering quantities. More details can be found in Section 7.

This paper contributes to the literature in three ways. First, to the best of our knowledge, this paper is the first to study supply chain contract design with considerations of financial constraints of both parties (the supplier and the distributor), productivity yield uncertainty and demand uncertainty. Second, a BBRS coordination contract is proposed, which can perfectly coordinate the supply chain. In other words, under the BBRS, the profits and the optimal strategies are the same with those under the centralized case. Third, there is a built-in mechanism in the BBRS to allocate the spillover profit between the supplier and the distributor, so that both of them can improve their expected profits under the BBRS.

The rest of the paper is organized as follows. In Section 2, we review the related literature and discuss the difference between this paper and the existing literature. In Section 3, we describe notations and set up the model. Then, we discuss the decentralized case and derive the optimal solutions in Section 4. In Section 5, we derive the optimal strategy and profit under the centralized case. We propose a BBRS mechanism and derive the optimal strategies in Section 6. In this section, we also compare the solutions under the decentralized case, the centralized case with those under the BBRS. In Section 7, we give some illustrating numerical results and do some further discussions. We conclude the paper in Section 8.

2 Literature Review.

In this section, the related literature is reviewed from three aspects: capital constraints, productivity yield uncertainty, and supply chain coordination.

2.1 Capital Constraints.

Capital constraint is very common in supply chains, and it often causes a capital deficit, which can be a problem for the supplier and/or the distributor.

Some researchers focus mainly on the distributor's capital deficit with demand uncertainty. Among them, some consider the situation where the distributor applies for loans from financial institutions. See e.g. Buzacott and Zhang (2004), Dada and Hu (2008) and Yan and Sun (2015). Some others focus on comprehensive decisions in forms of trade credits from various aspects. Related papers include those on trade credit and comprehensive decision of inventory (Huang et al., 2010; Moussawi-Haidar and Jaber, 2013); trade credits and comprehensive decision of optimal ordering quantity (Lou and Wang, 2013b; Ouyang et al., 2009; Annadurai and Uthayakumar, 2012; Shin,et al., 2018); optimal production quantity decision with trade credits (Lou and Wang, 2013a; Teng et al., 2012); trade credit considered as an endogenous variable and motivation tool to study supply chain coordination (Bandaly et al., 2014; Chen and Wang, 2012; Lee and Rhee, 2011). Jing and Seidmann (2014) show that when the production cost is relatively low, the trade credit financing can reduce the double marginal effect more effectively than the bank loan financing and otherwise the bank loan financing performs better. According to Cai et al. (2014), complement exists in bank loan financing and trade credit financing, when the distributor's initial capital is at an extremely low level. Yan et al. (2016) design a partial credit guarantee contract for supply chain financing systems, incorporating the bank credit financing and the manufacturer's trade credit guarantee. However, none of those mentioned above consider the supply uncertainty faced by the capital constrained distributor. When the upstream supplier has yield uncertainty, the supply is uncertain, which increases the financing risk of the distributor. In this paper, we consider both the market demand uncertainty and the supplier's productivity yield uncertainty.

Some other researchers consider the capital insufficiency faced by the supplier with no productivity yield uncertainty. Lai et al. (2009) consider the effectiveness of the supply chain under the circumstances of the supplier's capital constraint in reservation, delegate, and mixed forms. Mateut and Zanchettin (2013) and Thangam (2012) consider the optimal price discount and batch-ordering policy of perishable goods in supply chains with advance payments.

Capital deficit, to some extent, may occur to both the supplier and the distributor in a supply chain at the same time. Raghavan and Mishra (2011) point out that a bank is also willing to grant loans to distributors as it does for suppliers. Kouvelis and Zhao (2012) analyze the decisions involved in optimally structuring the trade credit contract from the supplier's perspective, while the supplier needs to get a bank loan. They conclude that a risk-neutral supplier should always finance the distributor with a trade credit at rates less than or equal to the risk-free bank interest rate. Protopappa-Sieke and Seifert (2017) demonstrate that there are significant benefits when the members of the supply chain share the working capital. Unlike the above literature, in this paper, we consider the coordination strategy for capital constrained supply chains with yield uncertainty.

2.2 Productivity Yield Uncertainty.

Suppliers in supply chains may have a productivity yield uncertainty. For example, there are yield uncertainties in supply chains of the agricultural products due to the impacts of weather, and/or the farmer's skills and efforts. In addition, production systems with uncertain yield can be found in many other industries, such as coal industry, steel industry, chemical industry (e.g., for the production of special chemicals or tailor made chemicals) and electronics industry (e.g., for the production of special processors or silicon chips) (Hu et al., 2013; Peng et al., 2013; Caro et al., 2012).

There have been more and more researchers who concern the operation strategies under yield uncertainty. Chen and Xiao (2015) consider the backup sourcing strategy of the buyer and the production planning of the supplier in presence of both the yield uncertainty and the demand uncertainty. Kouvelis et al. (2018) propose two technical assumptions to ensure the unimodality of the objective functions in two classes of price and quantity decision problems with one procurement opportunity under supply random yield and deterministic demand in a price-setting environment. Nasr et al. (2017) consider an economic production quantity with imperfect items where the quality of items produced within the same production run is correlated. They investigate the impact of correlation on the system performance measures and draw insights in terms of correlation effects on the production and maintenance policies. Eskandarzadeh et al. (2016) and Peng and Pang (2019) analyze optimal strategies for a supply chain with random yield of production. They apply Conditional Value at Risk

(CVaR) measure to model the risk preferences of the producer.

There are also some researchers who consider the coordination contracts for the supply chain with yield uncertainty. Peng et al. (2013) consider the coordination models in the supply chain where there are uncertain two-echelon yields and random demand. They investigate three contracts, revenue sharing (RS), overproduction risk sharing (OS) and combination of RS and OS (RO), and compare them with uncoordinated models. Their results indicate that the RS contract and OS contract both have their advantages and disadvantages, and the RO contract works the best on the whole supply chain. Giri and Bardhan (2015) consider a two-echelon supply chain involving one manufacturer and one retailer for a single product. The manufacturer's production is subject to some yield uncertainty, and there is a possibility of supply disruption in which no item from the supplier can reach the retailer. They propose a contract to coordinate the supply chain and find threshold conditions for which the coordinated model would collapse.

Suppliers with uncertain yield are usually small and medium enterprises, and they typically face capital insufficiency. When a supplier faces an uncertain yield, the bank (or the creditor) who lends money to the supplier may face a risk of the supplier bankruptcy when the yield is not as strong as expected. Because of the bankruptcy risk, it can be very difficult for the supplier to get loans from banks or other financial institutions. None of the above mentioned papers consider the financing issue of supply chains with productivity yield uncertainty, which is the main focus of this paper. Actually, we show that the financing problem can be solved by a BBRS coordination contract in this paper.

2.3 Supply Chain Coordination.

In the existing literature, supply chain contracts are studied without concerns of cash flow constraints or worries of bankruptcy, except that the recent works by Kouvelis and Zhao (2016) and Xiao et al. (2017). They consider three coordination contracts: revenue-sharing, buyback and quantity discount in a capital-constrained supply chain. Their results indicate that the coordination contracts cannot coordinate the financially constrained supply chain or can only coordinate the financially constrained supply chain under particular conditions. Kouvelis and Zhao (2016) show that with only variable default costs, buyback contracts remain coordinating and equivalent to revenue sharing contracts but are Pareto dominated by revenue-sharing contracts when fixed default costs are present.

Buyback contracts are commonly used in practice and are among the most well-studied supply chain contracts in literature. Typically, a buyback contract specifies that the supplier buys back any unsold inventory for some agreed-upon buyback price (Dai et al., 2012). Zhao et al. (2014) explore buyback contracts in a supplier-retailer supply chain where the retailer faces a price-dependent downward-sloping demand curve subject to uncertainty. Chen et al. (2017) investigate the combined impacts of fairness concerns and buyback guarantee financing on two members' equilibrium strategies and supply chain performance where the retailer has capital constraint. Zhang et al. (2016) examine differences in the performance of buyback and revenue-sharing contracts when suppliers have the authority to set contract parameters. They find that revenue-sharing contracts are more profitable for the supplier than buyback contracts in a high critical ratio environment when accounting for the supplier's parameter-specification behavior.

Mutual subsidy contract is another type of coordination contract which may work well. Peng and Pang (2018) consider a seasonal product supply chain channel over a period consisting of a low season and a high season, and they propose a mutual subsidy mechanism, in order to encourage the supplier to supply more raw material during the high season and to encourage the manufacturer to order more raw material during the low season.

However, the traditional coordination contracts sometimes fail to coordinate the supply chain with capital constraints or yield uncertainty. Xiao et al. (2017) show that the all-unit quantity discount contract fails to coordinate a financially constrained supply chain. However, the revenue-sharing and buyback contracts can coordinate the supply chain, when the supply chain has a sufficient total working capital. Luo and Chen (2016) find that the traditional revenue sharing contract cannot coordinate the supply chain with random yield and stochastic demand, but it would work if combined with a surplus subsidy mechanism. Peng et al. (2018) show that the quantity discount contract can efficiently coordinate the low-carbon supply chain with yield uncertainty, but the revenue-sharing contract cannot. A new contract of revenue-sharing with subsidy on emission reduction (RSS) is designed to coordinate the supply chain.

To the best of our knowledge, no one has ever considered the coordination for supply chains with financial constraints of both parties (the supplier and the distributor), productivity yield uncertainty and demand uncertainty. In this paper, we propose a BBRS contract, which can coordinate the supply chain. What differentiates our paper with the existing literature is that we integrate the risk sharing feature into our model. In addition, there is a built-in feature to allocate the spillover profit.

3 Model Formulation.

We consider a supply chain consisting of a supplier and a distributor, in which both have capital constraints. We assume that the supplier faces productivity yield uncertainty and the distributor faces market demand uncertainty. For convenience, subscript s is used for the supplier, and subscript d is used for the distributor.

We assume that, per unit of the product, the production cost is c, the wholesale price is w and the product retail price is p. We also assume that the product scrap value after the end of the sales period is zero, and both the supplier and the distributor are risk neutral.

In this paper, the wholesale price w and the retail market price p both are assumed to be exogenous and fixed values. Typically, the wholesale price w and the retail price p should be determined by the total supply and demand. In this paper, we focus on the supply and demand in a single supply chain instead of the whole supply and demand of the product. Suppliers with random yields are usually small and medium enterprises in vulnerable and weak competitive positions. The yield uncertainty has almost no impact on the total supply in the whole industry. Therefore, most of the suppliers with random yields have very little say in price decision and they have to sell the products at the market price or a given price. For example, due to the market competitions, the farmers usually take the given market wholesale price and the retailers usually sell the agricultural products at the market price. Neither of them can freely choose the wholesale prices or the market prices. Similarly, in some small-scale industry of semiconductor and electronics, the wholesale price cannot be freely chosen. Further, because the yield of small and medium enterprise has a small proposition of products, their yield fluctuations barely affect prices. For this reason, many researchers who study supply chain management with random yields consider the wholesale price and the retail price as fixed values (Hu et al., 2013; Caro et al., 2012; Chen and Xiao, 2015; Li and Li, 2016; Luo and Chen, 2016; Nasr et al., 2017). So, we consider the wholesale price w and the retail market price p both to be exogenous in this paper.

The following notations are used in this paper:

X :	the supplier's productivity yield random variable ($\mathbf{E}[X] = 1$);
$f(\cdot)$:	the probability density function of X;
$F(\cdot)$:	the cumulative distribution function of X;
$\overline{F}(\cdot)$:	the survival function of X;
Y :	the demand random variable ($\mathbf{E}[Y] = D$);
$g(\cdot)$:	the probability density function of Y;
$G(\cdot)$:	the cumulative distribution function of Y;
$\overline{G}(\cdot)$:	the survival function of Y;
ζ_s, ζ_d :	initial capitals of the supplier and the distributor;
q_d :	the distributor's ordering quantity;
q_s :	the supplier's planned production quantity;
T_0 :	the beginning time of the production period;
T_1 :	the end of the production period/the start of the distribution period;
T_2 :	the end of the distribution period;
r_p :	the effective risk free interest rate for the production period $[T_0, T_1]$;
$\tilde{r}_p$:	the nominal interest rate for the production period $[T_0, T_1]$;
r_d :	the effective risk free interest rate for the distribution period $[T_1, T_2]$;
$\tilde{r}_d$:	the nominal interest rate for the distribution period $[T_1, T_2]$;
$\tilde{r}_{pd}$:	the nominal interest rate for the whole circle $[T_0, T_2]$;
c_r :	the order shortage penalty per unit for the supplier;
c_h :	the inventory holding cost per unit for the supplier.

Here we assume that both X and Y are positive continuous type random variables. For convenience, we further assume that $\mathbf{E}[X] = 1$. Our results still hold for positive random variables X with other expected values.

We consider the supply chain over the whole cycle $[T_0, T_2]$. At the beginning of the production period, T_0, the distributor signs a contract with the supplier to place an order of q_d and then, based on the order, the supplier makes a decision on the production quantity q_s. We assume that the supplier's capital deficit at this time can be financed by a bank loan with nominal interest rates $\tilde{r}_p$ during the production period $[T_0, T_1]$, or $\tilde{r}_{pd}$ over the whole period $[T_0, T_2]$. At the same time, the initial capital ζ_d of the distributor, and the supplier's capital surplus, if any, is deposited to a bank to earn risk-free interest with an effective rate of r_p over $[T_0, T_1]$. We assume that if the supplier cannot fulfill the distributor's order, the supplier faces a shortage penalty, which is denoted by c_r per unit shortage.

At the end of the production period, T_1, due to the production uncertainty, the supplier's actual production is $q_s X$. The supplier delivers the products to the distributor and collect the payment from the distributor according to the contract. The distributor may have some capital deficit, which is financed by a bank loan with a nominal interest rate of $\tilde{r}_d$

over the distribution period $[T_1, T_2]$. At the same time, if the distributor has any capital surplus, it is deposited into a bank to earn a risk-free interest with an effective rate of r_d over $[T_1, T_2]$.

The distributor's decision variable is q_d and the supplier's decision variable is q_s. The distributor and the supplier need to choose optimal q_d and q_s respectively, to maximize their expected profits at T_2.

We want to point out that, as the initial capitals are given, maximizing the expected profits at T_2 is equivalent to maximizing the expected terminal cash flow at T_2. In this paper, we may use both terminal cash flows and terminal profits interchangeably for convenience.

We assume that the nominal rates $\tilde{r}_d, \tilde{r}_d$ and $\tilde{r}_{pd}$ are determined by the required effective risk free rates r_p, r_d and the bankruptcy risks due to productivity yield uncertainty and market demand uncertainty. Further, to avoid trivial cases, we assume that $p > (1 + r_d)w$, and $w > (1 + r_p)c$. It is obvious that $p > (1 + r_d)(1 + r_p)c$.

We discuss three cases: the decentralized case, the centralized case and the BuyBack and Risk-Sharing (BBRS) case. Under different cases, the cash flows can be different at each of the time points T_0, T_1 and T_2. Details are given in sections 4-7. .

4 The Decentralized Supply Chain.

Now we consider the situation that capital deficits of the supplier and the distributor can only be financed by bank loans and there is no coordination between the supplier and the distributor. We use subscript 1 for decision variables and profit variables under this situation. Now we have a supply chain financing system that consists of a supplier, a distributor and a bank. This can be treated as a sequential game problem. First, the distributor chooses the order quantity q_{d_1} and then based on q_{d_1}, the supplier chooses the optimal production quantity q_{s_1}. Finally, based on the strategies of the supply chain, the bank decides the interest rate for the supplier and the distributor.

We assume that the bank's interest rates for the loans to the supplier and the distributor are $\tilde{r}_{p_1}$ and $\tilde{r}_{d_1}$ respectively. We further assume that the distributor knows the optimal strategy of the supplier and chooses q_{d_1} accordingly. Similarly, we assume that the supplier knows the bank's strategy of choosing the nominal rate, and chooses q_{s_1} based on that. The problem can be solved backward.

As the first step, we figure out the interest rate decision for the bank. We assume that the capital market is perfect (no taxes, transaction costs or bankruptcy costs) and all bank loans are competitively priced (perfectly competitive banking sector). The assumptions imply that the bank's interest rates $\tilde{r}_{p_1}$ and $\tilde{r}_{d_1}$ are chosen so that the bank is indifferent between issuing loans to the supplier (or the distributor) and earning the risk-free rates r_p, r_d (Kouvelis and Zhao, 2012, 2016). Due to the yield and demand uncertainty, there are bankruptcy risks with the supplier and the distributor. For this reason, the bank charges nominal rates of $\tilde{r}_{p_1}$ and $\tilde{r}_{d_1}$, which are usually larger than r_p and r_d, respectively (for risk compensation). Therefore, the nominal rates $\tilde{r}_{p_1}$ and $\tilde{r}_{d_1}$ are respectively determined by

$$\begin{aligned}(1+r_p)\eta_{s_1} &= \mathbf{E}\Big[\min\big\{w\min(q_{d_1},q_{s_1}X)-c_r(q_{d_1}-q_{s_1}X)^+-c_h(q_{s_1}X-q_{d_1})^+,\\ &\qquad (1+\tilde{r}_{p_1})\eta_{s_1}\big\}\Big], \quad (1)\\ (1+r_d)\eta_{d_1} &= \mathbf{E}\Big[\min\big\{p\min(q_{d_1},q_{s_1}X,Y)-c_h[\min(q_{d_1},q^*_{s_1}X)-Y]^+,(1+\tilde{r}_{d_1})\eta_{d_1}\big\}\Big], \quad (2)\end{aligned}$$

where $w\min(q_{d_1},q_{s_1}X), c_r(q_{d_1}-q_{s_1}X)^+$ and $c_h(q_{s_1}X-q_{d_1})^+$ are the supplier's sales income, shortage penalty, and inventory holding cost, respectively; $p\min(q_{d_1},q_{s_1}X,Y)$ and $c_h[\min(q_{d_1},q^*_{s_1}X)-Y]^+$ are the distributor's sales income and inventory holding cost; η_{s_1} is the bank loan amount of the supplier and η_{d_1} is the bank loan amount of the distributor. In particular, η_{s_1} and η_{d_1} are given by

$$\eta_{s_1}=(cq_{s_1}-\zeta_s)^+, \quad \eta_{d_1}=[w\min(q_{s_1}X,q_{d_1})-c_r(q_{d_1}-q_{s_1}X)^+-(1+r_p)\zeta_d]^+. \quad (3)$$

4.1 The Supplier's Optimal Strategy.

Next we consider the optimization problem faced by the supplier. Due to the capital constraint, the supplier needs to apply for a bank loan with the amount of $(cq_{s_1}-\zeta_s)^+$. We consider the expected profit at the end of the distribution period (T_2). The supplier's expected terminal cash flow at time T_2 is

$$\begin{aligned}\pi_{s_1}(q_{s_1};q_{d_1}) &= (1+r_d)\big[\mathbf{E}[w\min(q_{d_1},q_{s_1}X)-c_r(q_{d_1}-q_{s_1}X)^+-c_h(q_{s_1}X-q_{d_1})^+\\ &\quad -(1+\tilde{r}_{p_1})(cq_{s_1}-\zeta_s)^+]^+ +(1+r_p)(\zeta_s-cq_{s_1})^+\big], \quad (4)\end{aligned}$$

where $[w\min(q_{d_1},q_{s_1}X)-c_r(q_{d_1}-q_{s_1}X)^+-c_h(q_{s_1}X-q_{d_1})^+-(1+\tilde{r}_{p_1})(cq_{s_1}-\zeta_s)]^+$ is the supplier's terminal cash flow if the initial capital is not enough to cover the production cost, and $(1+r_p)(\zeta_s-cq_{s_1})^+$ is the deposit income if the initial capital is more than the production cost.

If the yield uncertainty results in a low revenue such that the supplier cannot repay the principal and interest of the bank loan, the supplier goes bankrupt, and the supplier's residual equity becomes zero.

According to the standard backward induction, the bank's decision on the loan interest rate (see (1)) influences the supplier's decision. The decision model for the supplier is:

$$\max_{q_{s_1}} \pi_{s_1}(q_{s_1};q_{d_1}), \qquad \text{subject to} \quad (1). \quad (5)$$

By virtue of (1), (3) and noting that $[x-a]^+ = x - \min(x,a)$, we can rewrite (4) as

$$\begin{aligned}
\pi_{s_1}(q_{s_1};q_{d_1}) &= (1+r_d)\Big\{\mathbf{E}[(w+c_r+c_h)\min(q_{d_1},q_{s_1}X) - c_h q_{s_1}X] + (1+r_p)(\zeta_s - cq_{s_1})^+ \\
&\quad - c_r q_{d_1} - \mathbf{E}[\min\{w\min(q_{d_1},q_{s_1}X) - c_r(q_{d_1}-q_{s_1}X)^+ - c_h(q_{s_1}X - q_{d_1})^+, \\
&\qquad (1+\tilde{r}_{p_1})(cq_{s_1}-\zeta_s)^+\}]\Big\} \\
&= (1+r_d)\big\{\mathbf{E}[(w+c_r+c_h)\min(q_{d_1},q_{s_1}X)] \\
&\qquad - (1+r_p)(cq_{s_1}-\zeta_s) - c_r q_{d_1} - c_h q_{s_1}\big\} \\
&= (1+r_d)\Bigg[(w+c_r+c_h)\int_0^{q_{d_1}} \overline{F}\left(\frac{x}{q_{s_1}}\right)dx \\
&\qquad - (1+r_p)(cq_{s_1}-\zeta_s) - c_r q_{d_1} - c_h q_{s_1}\Bigg]. \qquad (6)
\end{aligned}$$

We have the following result:

Proposition 1 *Under the decentralized case, for any given order quantity q_{d_1}, the supplier's optimal production quantity $q^*_{s_1}$ is given by*

$$q^*_{s_1} = k_1 q_{d_1}, \qquad (7)$$

where k_1 satisfies

$$\int_0^{\frac{1}{k_1}} xf(x)dx = \frac{(1+r_p)c + c_h}{w+c_r+c_h}. \qquad (8)$$

Proof of Proposition 1 is in Appendix A.1.

In addition, by virtue of (6), (7) and (8), we can derive the maximal terminal cash flow for the supplier.

Proposition 2 *Under the decentralized case, for any given order quantity q_{d_1}, the supplier's maximal terminal cash flow is given by*

$$\pi^*_{s_1} = (1+r_d)\left[(w+c_r+c_h)q_{d_1}\overline{F}\left(\frac{1}{k_1}\right) + (1+r_p)\zeta_s - c_r q_{d_1}\right]. \qquad (9)$$

4.2 The Distributor's Optimal Strategy.

Now we consider the optimization problem faced by the distributor. The expected terminal cash flow function of the distributor is

$$\begin{aligned}
\pi_{d_1}(q_{d_1};q_{s_1}) &= \mathbf{E}\Bigg[[p\min(q_{d_1},q_{s_1}X,Y) - c_h[\min(q_{d_1},q^*_{s_1}X) - Y]^+ - (1+\tilde{r}_{d_1})\eta_{d_1}]^+ \\
&\qquad + (1+r_d)[(1+r_p)\zeta_d - w\min(q_{d_1},q_{s_1}X) + c_r(q_{d_1}-q_{s_1}X)^+]^+\Bigg], \qquad (10)
\end{aligned}$$

where η_{d_1} is given by (3), $p\min(q_{d_1},q_{s_1}X,Y)$ is the sales revenue and $w\min(q_{d_1},q_{s_1}X)$ is the wholesale cost and $(1+r_d)[(1+r_p)\zeta_d - w\min(q_{d_1},q_{s_1}X) + c_r(q_{d_1}-q_{s_1}X)^+]^+$ is the deposited income if the capital is more than the wholesale cost. If the demand uncertainty results in a bad revenue such that the distributor cannot repay the principal and interest of the bank loan, the distributor goes bankrupt, and the distributor's residual asset becomes zero.

We assume that the distributor makes the decision (choosing the optimal q_{d_1}) based on the assumption that, for any given ordering quantity q_{d_1}, the supplier chooses the optimal planned production quantity $q_{s_1}^*$ given by (7) and the bank chooses the optimal nominal interest rate $\tilde{r}_{d_1}$ given by (2). So the decision model for the distributor is:

$$\max_{q_{d_1}} \pi_{d_1}(q_{d_1}; q_{s_1}), \qquad \text{subject to} \quad (7) \quad \text{and} \quad (2). \tag{11}$$

By virtue of (2) and (7) we can rewrite (10) as

$$\begin{aligned}
\pi_{d_1}(q_{d_1}; q_{s_1}^*) &= \mathbf{E}\{p\min(q_{d_1}, q_{s_1}^* X, Y) - c_h[\min(q_{d_1}, q_{s_1}^* X) - Y]^+\} \\
&\quad -(1+r_d)\mathbf{E}[(w+c_r)\min(q_{d_1}, q_{s_1}^* X) - c_r q_{d_1} - (1+r_p)\zeta_d] \\
&= (p+c_h)\int_0^{q_{d_1}} \overline{G}(x)\overline{F}\left(\frac{x}{k_1 q_{d_1}}\right) dx - c_h \int_0^{q_{d_1}} \overline{F}\left(\frac{x}{q_{s_1}}\right) dx \\
&\quad -(1+r_d)\left[(w+c_r)\int_0^{q_{d_1}} \overline{F}\left(\frac{x}{k_1 q_{d_1}}\right) dx - c_r q_{d_1} - (1+r_p)\zeta_d\right].
\end{aligned} \tag{12}$$

We have the following result:

Proposition 3 *Under the decentralized case, the distributor's optimal ordering quantity* $q_{d_1}^*$ *is given by*

$$\begin{aligned}
&k_1 \int_0^{\frac{1}{k_1}} [(p+c_h)\overline{G}(k_1 q_{d_1}^* x) - (1+r_d)(w+c_r)] x f(x) dx \\
&+\overline{F}\left(\frac{1}{k_1}\right)\left[(p+c_h)\overline{G}(q_{d_1}^*) - (1+r_d)(w+c_r - c_h)\right] + (1+r_d)c_r = 0
\end{aligned} \tag{13}$$

Proof of Proposition 3 can be found in Appendix A.2.

Moreover, the distributor's optimal terminal cash flow is given by the following proposition:

Proposition 4 *Under the decentralized case, the distributor's optimal terminal cash flow is*

$$\begin{aligned}
\pi_{d_1}^* &= (p+c_h)q_{d_1}^*\left[k_1 \int_0^{\frac{1}{k_1}} \overline{G}(k_1 q_{d_1}^* x)[\overline{F}(x) - x f(x)]dx - \overline{G}(q_{d_1}^*)\overline{F}\left(\frac{1}{k_1}\right)\right] \\
&\quad -c_h k_1 q_{d_1}^* \int_0^{\frac{1}{k_1}} x f(x) dx + (1+r_p)(1+r_d)\zeta_d,
\end{aligned} \tag{14}$$

where k_1 *is given by* (8) *and* $q_{d_1}^*$ *is given by* (13).

5 The Centralized Supply Chain.

Now we consider the centralized case. When the supply chain is centralized, the optimal solution gives us the maximal possible expected total profit of the whole supply chain. Later in Section 6 we propose a coordination mechanism to achieve this maximal possible expected total profit for the whole supply chain.

We use subscript 0 for decision variables and profit variables of the centralized supply chain. The centralized supply chain system has the working capital $\zeta_s + \zeta_d$. Therefore, the nominal rate $\tilde{r}_{pd_0}$ for the whole period is determined by

$$(1+r_p)(1+r_d)\eta_c = \mathbf{E}[\min\{p\min(q_{s_0}X,Y) - c_h(q_{s_1}X-Y)^+], (1+\tilde{r}_{pd_0})\eta_c\}], \tag{15}$$

where $\eta_c = (cq_{s_0} - \zeta_s - \zeta_d)^+$. By virtue of the formula $(x-a)^+ = x - \min(x,a)$ and (15), we can get the expected total terminal cash flow as

$$\begin{aligned}
\pi_{c_0}(q_{s_0}) &= \mathbf{E}[(p+c_h)\min(q_{s_0}X,Y) - c_h q_{s_0}X] \\
&\quad -(1+\tilde{r}_{pd_0})\eta_c]^+ - (1+r_p)(1+r_d)(\zeta_s+\zeta_d-cq_{s_0})^+ \\
&= \mathbf{E}[(p+c_h)\min(q_{s_0}X,Y) - c_h q_{s_0}X] - (1+r_p)(1+r_d)(cq_{s_0} - \zeta_s - \zeta_d) \\
&= (p+c_h)q_{s_0}\int_0^\infty \overline{G}(q_{s_0}x)\overline{F}(x)dx - c_h q_{s_0} \\
&\quad - (1+r_p)(1+r_d)(cq_{s_0} - \zeta_s - \zeta_d).
\end{aligned} \tag{16}$$

We have the following result:

Proposition 5 *Under the centralized case, the optimal production quantity $q_{s_0}^*$ of the centralized supply chain is given by*

$$(p+c_h)\int_0^\infty xf(x)\overline{G}(q_{s_0}^*x)dx = (1+r_p)(1+r_d)c + c_h. \tag{17}$$

Proof of Proposition 5 is in Appendix A.3.

Remark 1 *A general explicit formula for $k_1, q_{d_1}^*, q_{s_0}^*$ to solve* (8), (13), (17) *are not available. Therefore, we need to use some numerical methods, such as the Bi-Section method, or Newton's method. From* (8), (13), (17), *it is easy to see that the left hand sides are monotonous functions with respect to $k_1, q_{d_1}^*, q_{s_0}^*$. Therefore, the classical Bi-Section method should work well. Alternatively, some equation solver in Matlab or other software can be used to obtain the numerical solution of* (8), (13), (17).

In addition, it is easy to get that

Proposition 6 *The optimal terminal cash flow under the centralized case is given by*

$$\pi_{c_0}^* = (1+r_p)(1+r_d)(\zeta_s+\zeta_d) + (p+c_h)\int_0^\infty xg(x)\overline{F}\left(\frac{x}{q_{s_0}^*}\right)dx, \tag{18}$$

where $q_{s_0}^*$ is given by (17).

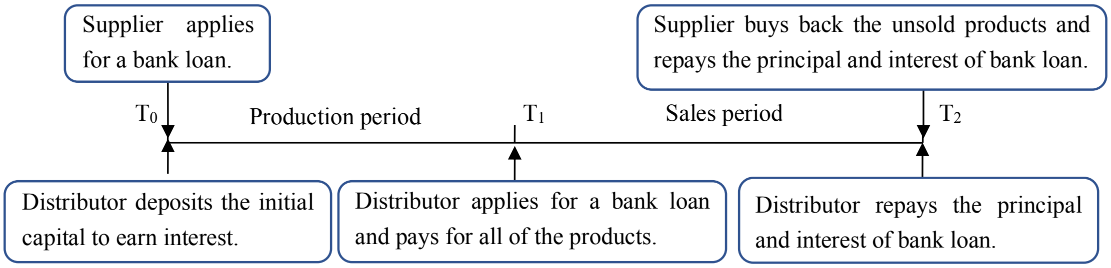

Figure 1: The sequence of events under the BBRS.

6 The Buyback and Risk Sharing Mechanism.

Usually, unless both the supplier and the distributor belong to the same company, the centralized supply chain cannot be implemented. Here we propose a coordination mechanism of *buyback and risk sharing* (BBRS), aiming to achieve the maximal possible expected total profits for the whole supply chain. Under the BBRS, not only the expected total profit of the whole supply chain can be maximized, but also there is a built-in feature to allocate the spillover profit between the supplier and the distributor. We use subscript 2 for variables under this BBRS.

Due to the productivity yield uncertainty, the supplier may produce more than the distributor's order quantity. This is also a risk for the supplier if the overproduced product cannot be sold. The BBRS mechanism assumes that, to share this overproduction risk, the distributor buys the overproduced products (the quantity is $(q_{s_2}X - q_{d_2})^+$) at the price of w. Moreover, due to the productivity yield uncertainty, the supplier may produce less than the distributor's order quantity and a penalty cost may apply. This is another risk for the supplier. We assume that the distributor also shares this risk by exempting the supplier from the penalty.

In addition, due to the market demand uncertainty, the distributor faces a risk that the ordered product may not be sold out completely. The BBRS mechanism assumes that, to share the risk of demand uncertainty, the supplier buys back the unsold products in excess of a certain level v_0, at the price of μ. Therefore, the buyback quantity is $(q_{s_2}X - Y)^+ - v_0$. μ and v_0 are parameters to be determined. See Figure 1 for the cash flows under the BBRS.

The value of μ determines the supply chain's total profit. The distributor can choose μ such that the total profit of the supply chain under this BBRS is the same as the total profit of the centralized supply chain. Intuitively, μ would be less than or equal to $(1+r_d)w$ in practice. Our results in this section do confirm this intuition.

A higher expected total profit does not always imply higher expected profits for both the supplier and the distributor at the same time. To ensure that both the supplier and the distributor to participate in the BBRS mechanism, both parties must have a profit that is no less that under the decentralized case. This is the reason we introduce the v_0 parameter. The choice of the parameter v_0 determines the allocation of the spillover profit, to ensure that both parties are better off by participating in the BBRS mechanism. Here, we do not require that v_0 to be positive. Sometimes, the parameter v_0 may be negative, which means that the supplier compensate the distributor beyond the unsold products.

6.1 Cash Flows and Nominal Interest Rates Under the BBRS.

Under the BBRS, at the end of the production period T_1, the distributor's payment to the supplier is

$$C_{T_1} \equiv wq_{s_2}X, \tag{19}$$

and the payment of the supplier to the distributor at T_2 is

$$C_{T_2} \equiv \mu(q_{s_2}X - Y)^+ - \mu v_0. \tag{20}$$

Denote $\eta_{s_2} = \zeta_s - cq_{s_2}$. Then $\eta_{s_2}^- = (\zeta_s - cq_{s_2})^- = (cq_{s_2} - \zeta_s)^+$ and it is the supplier's capital deficit at T_0, which is to be financed by a bank loan over $[T_0, T_2]$. If $\eta_{s_2} > 0$, then the supplier does not have any deficit, and the amount of $\eta_{s_2}^+ = (\zeta_s - cq_{s_2})^+$ is deposited into the bank to earn interest. Let the nominal rate over $[T_0, T_2]$ be $\tilde{r}_{pd_2}$. Assume that the bank is risk-neural. Then the nominal rate $\tilde{r}_{pd_2}$ is determined by

$$(1 + r_p)(1 + r_d)\eta_{s_2}^- = \mathbf{E}[\min\{(1 + r_d)C_{T_1} - C_{T_2}, (1 + \tilde{r}_{pd_2})\eta_{s_2}^-\}]. \tag{21}$$

Now consider the distributor's situation. At T_0, the distributor can deposit the initial capital ζ_d to the bank to earn interest with an effective rate of r_p over $[T_0, T_1]$. Denote $\eta_{d_2} = (1 + r_p)\zeta_d - C_{T_1}$. Then $\eta_{d_2}^-$ is the capital deficit of the distributor at T_1. Assume that the deficit is financed by a bank loan over $[T_1, T_2]$ with a nominal rate of $\tilde{r}_{d_2}$. For a bank, the nominal rate $\tilde{r}_{d_2}$ is determined by

$$(1 + r_d)\eta_{d_2}^- = \mathbf{E}[\min\{p\min(q_{s_2}X, Y) - c_h(q_{s_1}X - Y)^+ + C_{T_2}, (1 + \tilde{r}_{d_2})\eta_{d_2}^-\}]. \tag{22}$$

Under the BBRS, the distributor is the leader and the supplier is the follower. Before the production season, the supplier and the distributor engage in negotiations to determine the contract parameters (μ, v_0). We drive the optimal strategy for the supplier first, then we derive the optimal strategy for the distributor.

6.2 The Supplier's Optimal Strategy Under the BBRS.

The expected terminal cash flow of the supplier at T_2 under the BBRS is

$$\pi_{s_2}(q_{s_2}; q_{d_2}) = \mathbf{E}\big[[(1 + r_d)C_{T_1} - C_{T_2} - (1 + \tilde{r}_{pd_2})\eta_{s_2}^-]^+\big] + (1 + r_p)(1 + r_d)\eta_{s_2}^+. \tag{23}$$

By virtue of the formula $(x - a)^+ = x - \min(x, a)$ and (21), we can get that

$$\begin{aligned}
\pi_{s_2}(q_{s_2}; q_{d_2}) &= \mathbf{E}[(1 + r_d)C_{T_1} - C_{T_2}] - (1 + r_p)(1 + r_d)(\eta_{s_2}^- - \eta_{s_2}^+) \\
&= \mathbf{E}\big[(1 + r_d)wq_{s_2}X - \mu(q_{s_2}X - Y)^+\big] + \mu v_0 - (1 + r_p)(1 + r_d)(cq_{s_2} - \zeta_s) \\
&= (1 + r_d)wq_{s_2} - \mu\int_0^\infty \int_0^{q_{s_2}x} (q_{s_2}x - y)g(y)f(x)dydx + \mu v_0 \\
&\quad - (1 + r_p)(1 + r_d)(cq_{s_2} - \zeta_s).
\end{aligned} \tag{24}$$

We have the following result:

Proposition 7 *Under the BBRS, for any given ordering quantity q_{d_2}, the supplier's optimal production quantity $q_{s_2}^*$ is given by:*

$$\mu\int_0^\infty xG(q_{s_2}x)f(x)dx = (1 + r_d)[w - (1 + r_p)c]. \tag{25}$$

Proof of Proposition 7 is in Appendix A.4.

6.3 The Coordination Parameter μ.

The BBRS mechanism is proposed to improve the performance of the supply chain as well as the profits of both the supplier and the distributor. It turns out that the parameter of μ can be chosen such that the total profit of the supply chain can achieve the maximal possible profit, which is the total profit under the centralized case. In addition, the choice of v_0 determines the allocation of the spillover profit between the supplier and the distributor.

In this subsection, we show that μ can be chosen (see (26)) such that the supplier's optimal production quantity $q^*_{s_2}$ is the same as that under the centralized case $q^*_{s_0}$, and the expected total profit of the supply chain is the same as that under the centralized case. Finally, in subsection 6.4, we discuss the effect of v_0 and the allocation of the spillover profit.

By virtue of (25) and (17), we can get the following proposition:

Proposition 8 *Under the BBRS, if the μ is chosen as*

$$\mu = \frac{(1+r_d)(p+c_h)[w-(1+r_p)c]}{p-(1+r_p)(1+r_d)c}, \tag{26}$$

then the supplier's planned production quantity is equal to that under the centralized case, i.e. $q^*_{s_0} = q^*_{s_2}$.

It means that under the BBRS, when the distributor buys all the products from the supplier at the wholesale price of w, and the supplier buys back the unsold products at the price of μ given by (26), the production and sales quantities are the same with those under the centralized case.

By (26) and (25), we can get that the optimal production quantity $q^*_{s_2}$ under the BBRS satisfies

$$(p+c_h)\int_0^\infty xf(x)\overline{G}(q^*_{s_2}x)dx = (1+r_p)(1+r_d)c. \tag{27}$$

Therefore, by virtue of (24), we can get the supplier's expected cash flow as the follows

$$\begin{aligned}
\pi^*_{s_2}(q^*_{s_2}) &= \mathbf{E}\left[(1+r_d)wq^*_{s_2}X - \mu(q^*_{s_2}X-Y)^+\right] + (1+r_p)(1+r_d)(\zeta_s - cq^*_{s_2}) + \mu v_0 \\
&= (1+r_d)wq^*_{s_2} - \mu\int_0^\infty\int_0^{q_{s_2}x}(q^*_{s_2}x-y)g(y)f(x)dydx \\
&\quad + (1+r_p)(1+r_d)(\zeta_s - cq^*_{s_2}) + \mu v_0 \\
&= \mu\int_0^\infty\int_0^{q^*_{s_2}x} yg(y)f(x)dydx + (1+r_p)(1+r_d)\zeta_s + \mu v_0,
\end{aligned} \tag{28}$$

where μ is given by (26) and $q^*_{s_2}$ is given by (27).

Remark 2 *As we can tell from* (28)*, the supplier's expected optimal profit actually does not depend on the ordering quantity q_d under the BBRS. This is not surprising, because all the supplier's produced products are taken by the distributor with the wholesale price w under the BBRS. Further, from Proposition 8, we can get that*

$$\mu = (1+r_d)w - \frac{(1+r_p)(1+r_d)c[p+c_h-(1+r_d)w]}{p+c_h-(1+r_p)(1+r_d)c} < (1+r_d)w.$$

Now we consider the distributor's profit under the BBRS. In this case, by virtue of the formula $(x-a)^+ = x - \min(x,a)$ and (22), we can get the distributor's expected terminal cash flow as

$$
\begin{aligned}
\pi_{d_2}(q_{d_2}; q_{s_2}) &= \mathbf{E}\Big[\big[p\min(q_{s_2}X, Y) + C_{T_2} - c_h(q_{s_1}X - Y)^+ - (1+\tilde{r}_{d_2})\eta_{d_2}^-\big]^+ + (1+r_d)\eta_{d_2}^+\Big] \\
&= \mathbf{E}\Big[p\min(q_{s_2}X, Y) + C_{T_2} - c_h(q_{s_1}X - Y)^+ - (1+r_d)\eta_{d_2}^- + (1+r_d)\eta_{d_2}^+\Big] \\
&= \mathbf{E}\Big[p\min(q_{s_2}X, Y) + (\mu - c_h)(q_{s_2}X - Y)^+ - \mu v_0 \\
&\qquad + (1+r_d)[(1+r_p)\zeta_d - wq_{s_2}X]\Big]. \qquad (29)
\end{aligned}
$$

On the right hand side of the above equation, $p\min(q_{s_2}X, Y)$ is the expected revenue by seling the products, and $\mu[(q_{s_2}X - Y)^+ - v_0]$ is the risk compensation of the unsold products from the supplier.

Plugging (21) and (22) into (29), we can get that

$$
\begin{aligned}
\pi_{d_2}(q_{d_2}; q_{s_2}) &= \mathbf{E}\Big[p\min(q_{s_2}X, Y) + (\mu - c_h)(q_{s_2}X - Y)^+ \\
&\qquad - \mu v_0 - (1+r_d)[wq_{s_2}X - (1+r_p)\zeta_d]\Big]. \qquad (30)
\end{aligned}
$$

From (30), we can see that under the BBRS, the ordering quantity has no impact on the expected terminal cash flow of the distributor.

If μ is given by (26), and $q_{s_2}^*$ is given by (27), we can get that $q_{s_2}^* = q_{s_0}^*$. Then, by virtue of (16), (24) and (30), it is not hard to show that

$$
\pi_{c_0}^*(q_{s_0}^*) = \pi_{s_2}^*(q_{s_2}^*) + \pi_{d_2}^*(q_{d_2}; q_{s_2}^*). \qquad (31)
$$

Therefore, the total profit of the supply chain under the BBRS is equal to the total maximized profit for the centralized supply chain.

Next, we discuss the effect of the parameter v_0, which determines the allocation of the spillover profit between the supplier and the distributor.

6.4 The Coordination Parameter v_0.

We have showed that when μ is given in (26), and the planned production quantity is given by (27), the total profit is the same as the total profit in the centralized supply chain case. However, even the total profit is maximized, there is no guarantee that the profits of the supplier and the distributor are both higher than the profits under the decentralized case.

In the proposed BBRS model, the parameter v_0 can be used to determine the allocation of the spillover profit. We assume that the proportion of the spillover profit allocated to the supplier is $\alpha (0 \le \alpha \le 1)$. That is $\pi_{s_2}^* = \pi_{s_1}^* + \alpha(\pi_{c_0}^* - \pi_{c_1}^*)$, where $\pi_{c_1}^* = \pi_{s_1}^* + \pi_{d_1}^*$ is the terminal cash flow of the supply chain under bank loan financing. By virtue of (28), we can get v_0 as follows.

$$\begin{aligned}
v_0 &= \frac{1}{\mu}\left[\pi^*_{s_2} - (1+r_p)(1+r_d)\zeta_s\right] - \int_0^\infty \int_0^{q^*_{s_2}x} yg(y)f(x)dydx \\
&= \frac{1}{\mu}\left[\pi^*_{s_1} + \alpha(\pi^*_{c_0} - \pi^*_{c_1}) - (1+r_p)(1+r_d)\zeta_s\right] - \int_0^\infty \int_0^{q^*_{s_2}x} yg(y)f(x)dydx \\
&= \frac{1}{\mu}\left[(1-\alpha)\pi^*_{s_1} + \alpha(\pi^*_{c_0} - \pi^*_{d_1}) - (1+r_p)(1+r_d)\zeta_s\right] \\
&\quad - \int_0^\infty \int_0^{q^*_{s_2}x} yg(y)f(x)dydx, \qquad (32)
\end{aligned}$$

where $\pi^*_{s_1}, \pi^*_{c_0}, \pi^*_{d_1}$ are given by (9), (18) and (14), respectively.

Now we consider some special cases. First, if $\alpha = 0$, all the spillover profit is allocated to the distributor, and v_0 is given by

$$\begin{aligned}
v_0 &= \frac{1}{\mu}\left[\pi^*_{s_1} - (1+r_p)(1+r_d)\zeta_s\right] - \int_0^\infty \int_0^{q^*_{s_2}x} yg(y)f(x)dydx \\
&= \frac{1}{\mu}(1+r_d)wq^*_{d_1}\overline{F}\left(\frac{1}{k_1}\right) - \int_0^\infty \int_0^{q^*_{s_2}x} yg(y)f(x)dydx. \qquad (33)
\end{aligned}$$

Secondly, when $\alpha = 1$, all the spillover profit is allocated to the supplier, and v_0 is

$$\begin{aligned}
v_0 &= \frac{1}{\mu}\left[\pi^*_{c_0} - \pi^*_{d_1} - (1+r_p)(1+r_d)\zeta_s\right] - \int_0^\infty \int_0^{q^*_{s_2}x} yg(y)f(x)dydx \\
&= \frac{p}{\mu}\int_0^\infty xg(x)\overline{F}\left(\frac{x}{q^*_{s_2}}\right)dx - \int_0^\infty \int_0^{q^*_{s_2}x} yg(y)f(x)dydx \\
&\quad - \frac{pq_{d_1}}{\mu}\left[k_1\int_0^{\frac{1}{k_1}} \overline{G}(k_1 q_{d_1}x)[\overline{F}(x) - xf(x)]dx - \overline{G}(q_{d_1})\overline{F}\left(\frac{1}{k_1}\right)\right]. \qquad (34)
\end{aligned}$$

The supplier and the distributor usually determine the coordination parameter v_0 under the principle that their profits are both higher than those under the decentralized case. In reality, the BBRS mechanism should benefit both parties, and there are usually some negotiations between the supplier and the distributor regarding the spillover profit allocation. In this case, the value of v_0 usually depends on the negotiation skills of the distributor and the supplier.

Remark 3 *From the above analysis, we can see that under the BBRS, the production strategy and the whole profit of the supply chain are the same with those under the centralized case. So, the BBRS mechanism can realize the optimization of the whole supply chain under financial constraints and yield uncertainty. Further, by negotiating the value of v_0, the supplier and the distributor can allocate the spillover profit under the BBRS, so that both of them obtain higher profits than that under the decentralized case.*

7 Discussions and Numerical Results.

In this section, we compare the decentralized case and the BBRS mechanism. Meanwhile, for illustration purpose, we present a numerical example here. In particular, we consider a

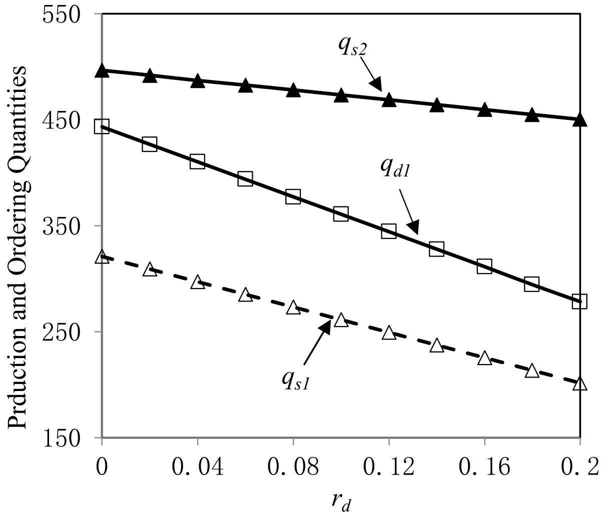

Figure 2: Production/ordering quantities vs. r_d

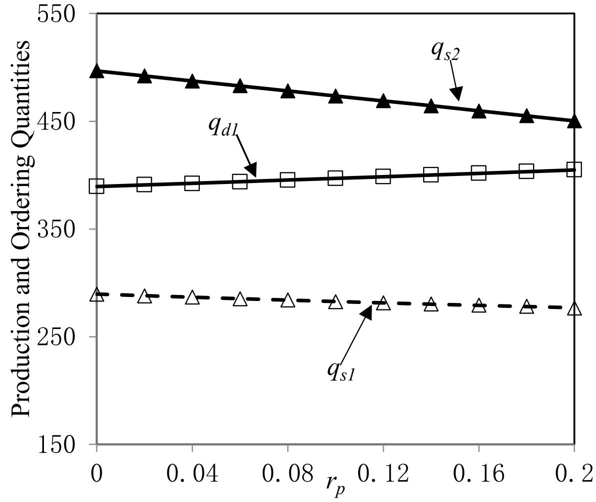

Figure 3: Production/ordering quantities vs. r_p

coal supply chain with a coal company (supplier) and a distributor, in which the product demand, Y, is uniformly distributed over $[0, 1000]$. The random variable of yield fluctuation, X, is uniformly distributed over $[0, 2]$. We choose the following values of parameters based on the data from Yanzhou Coal Industry in China in 2018. The retail price of clean coal is $p = 1000(CNY/ton)$ and the wholesale price is $w = 650(CNY/ton)$. The production cost is $c = 300(CNY/Ton)$ and the inventory holding cost is $c_h = 30(CNY/ton)$. We assume that the shortage cost $c_r = 50(CNY/ton)$, and the interest rates $r_p = r_d = 6\%$. In addition, we assume that the initial capital of the supplier (the coal company) $\zeta_s = 40000(CNY)$, and the initial capital of the distributor $\zeta_d = 20000(CNY)$. We further assume that the threshold of buyback $v_0 = 14(ton)$. The numerical results are presented in Figure 2 to Figure 9 and those results are discussed along with the qualitative analysis results.

7.1 Production and Ordering Quantities.

Proposition 9 *The production quantity under the BBRS (or under the centralized case) is higher than that under the decentralized case. That is*

$$q_{s_2}^*(= q_{s_0}^*) > q_{s_1}^*. \tag{35}$$

Proof of Proposition 9 is in Appendix A.5.

The numerical results regarding sensitivity of the production and ordering quantities (q_{s_1}, q_{s_2} and q_{d_1}) with respect to the interest rate r_d in the distribution period and the interest rate r_p in the production period are given in Figure 2 and Figure 3. In particular, in Figure 2, we fix r_p and investigate the sensitivities of q_{s_1}, q_{s_2} and q_{d_1} with respect to r_d. In Figure 3, we fix r_d and investigate the sensitivities of q_{s_1}, q_{s_2} and q_{d_1} with respect to r_p. Noting that the ordering quantity q_{d_2} under the BBRS case does not matter (see Proposition 8 and Remark 2), so it does not appear in Figure 2 and Figure 3.

From Figure 2, we can see that under the decentralized case, the production quantity q_{s_1} and ordering quantity q_{d_1} both decrease with respect to the expected interest rate r_d for

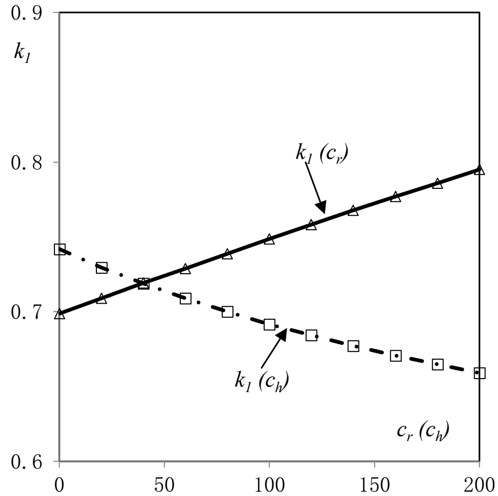

Figure 4: Production/ordering quantities vs. c_r

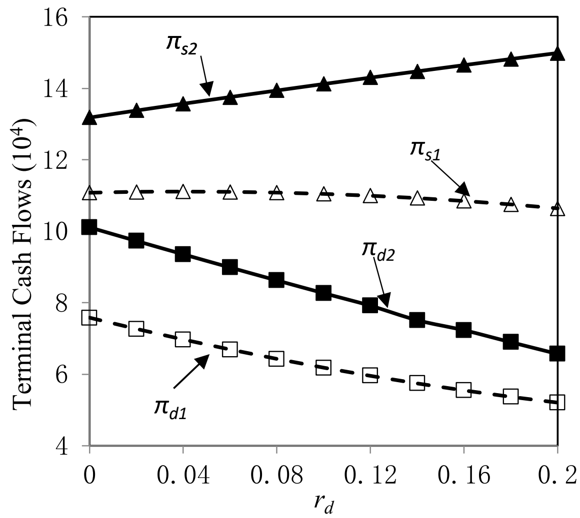

Figure 5: Terminal cash flows vs. r_d

the distribution period. It is not surprising. For a higher interest rate r_d, the distributor faces a higher financial burden. Therefore, the distributor tends to order less (smaller q_{d_1}) to decrease the demand uncertainty risk. In addition, when r_p is given, the production strategy k_1 is fixed (see (8)), therefore, the production quantity q_{s_1} decreases with respect to r_d.

From Figure 3, we can see that the production quantity q_{s_1} under the decentralized case decreases with respect to the expected interest rate for the production period r_p. However, the ordering quantity q_{d_1} increases with r_p. The reason is as follows. From (8), we can get that $\frac{dk_1}{dr_p} < 0$. When the ordering quantity q_{d_1} is given, the supplier's production quantity q_{s_1} decreases with r_p. Therefore, to motivate the supplier to produce more, the distributor tends to order more when r_p becomes larger.

There is a managerial insight based on the above analysis. For a certain supply chain, e.g., an agricultural supply chain, the government can subsidize the financing interest for the supplier to increase the production, and/or subsidize the financing interest for the distributor to increase the ordering quantity. Actually, it is very common in China that the government subsidizes the financial costs for agriculture related loans. Moreover, when subsidizing the supplier's financing interest, the distributor tends to decrease the ordering quantity, so some motivations are needed to make the distributors to order more products.

In addition, from Figures 2 and Figure 3, we can also see that the production quantity q_{s_2} under the BBRS decreases with r_d or r_p. Noting that the BBRS can achieve the level under the centralized case, and all the risks due to yield uncertainty and demand uncertainty are shared by the supplier and the distributor, a higher value of r_d or r_p increases the interest rate cost for the whole supply chain, and the supplier would choose a lower production level (smaller q_{s_2}) to reduce the total risks. Therefore, under the BBRS or the centralized case, subsidizing the financing interest either for the supplier or for the distributor (to reduce r_p or r_d) can improve the production quantity q_{s_2}.

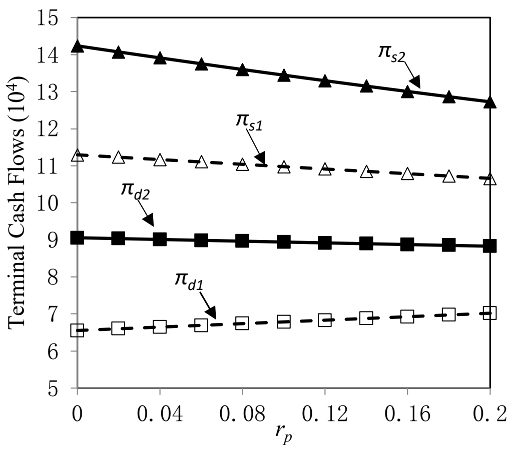

Figure 6: Terminal cash flows vs. r_p

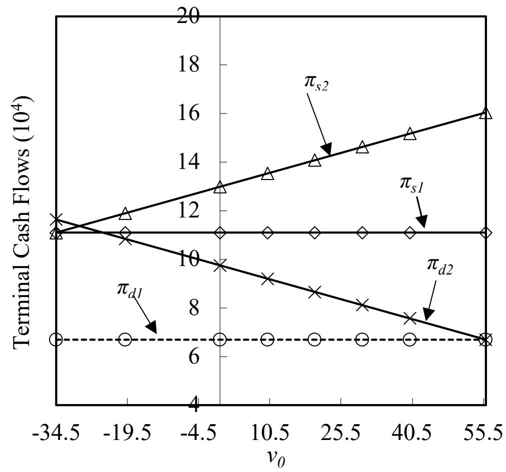

Figure 7: Terminal cash flows vs. v_0

From (8), we can get that

$$\int_0^{\frac{1}{k_1}} xf(x)dx = \frac{(1+r_p)c + c_h}{w + c_r + c_h} = 1 - \frac{w + c_r - (1+r_p)c}{w + c_r + c_h}. \tag{36}$$

Therefore, noting that $w > (1+r_p)c$, we have

$$\frac{dk_1}{dc_r} > 0, \quad \frac{dk_1}{dc_h} < 0.$$

The numerical results showed in Figure 4 are consistent with the above analytic results. The managerial insight is that, the supply shortage penalty can improve the supplier's production enthusiasm, while the inventory holding cost can reduce the supplier's production enthusiasm.

7.2 Profits.

From (17) and (27), we can see that, if we choose the parameter μ as given in (26), the production quantity under the BBRS is the same with that under the centralized case. Further, under the BBRS, all the supplier's products are purchased by the distributor at the wholesale price w. So, the whole profits of the supply chain must be the same with that under the centralized case.

Under the BBRS mechanism, by negotiating the coordination parameter v_0, the supplier and the distributor can allocate the spillover profit such that both of them can realize higher profits under the BBRS. Therefore, the BBRS mechanism can optimize the supply chain with financial constraints and yield uncertainty.

The numerical results on the dependence of the profits (reflected by the terminal cash flows), with respect to the interest rates r_d, r_p and the parameter v_0, are presented in Figures 5-7. Figure 5 shows that the profits of the supplier (π_{s_1}) and the distributor (π_{d_2}) under the decentralized case both decrease with the expected interest rate for the distribution period

r_d. This is because when r_d increases, the distributor's ordering quantity q_{d_1} decreases. Because the supplier's production strategy parameter k_1 ($q_{s_1} = k_1 q_{d_1}$) is independent of r_d (see (8)), the supplier's production quantity q_{s_1} decreases, too.

Figure 6 shows that, under the decentralized case, when the expected interest rate for the production period r_p increases, the supplier's profit (π_{s_1}) decreases, but the distributor's profit (π_{d_1}) increases. It is not surprising that the supplier's profit decreases as r_p increases, because a higher value of r_p means a higher financing cost for the supplier. The interesting part is that the distributor can benefit from the increasing financing cost (higher value of r_p) for the supplier. There could be two reasons behind that. First, the distributor is the game leader, and she knows that the supplier would response to the higher value of r_p by choose a lower value of k_1 (see (8)), so the distributor can choose the ordering quantity q_{d_1} to maximize her own profit. Second, the distributor earns interest with the rate of r_p on her initial capital ζ_d. So a higher value of r_p can benefit the distributor.

Now look at the supplier's profit π_{s_2}, and the distributor's profit π_{d_2} under the BBRS case. From Figure 6, we can see that, under the BBRS, the profits of both the supplier (π_{s_2}) and the distributor (π_{d_2}) decrease with respect to the production period interest rate r_p. However, from Figure 5, we can see that the distributor's profit π_{s_2} decreases, but the supplier's profit π_{s_2} increases with respect to r_d. The reason is as follows. Under the BBRS, from (26), we can get that

$$\mu = \frac{(p + c_h)[w - (1 + r_p)c]}{\frac{p}{1+r_d} - (1 + r_p)c}.$$

So, we can get that $\frac{d\mu}{dr_d} > 0$, which means that the amount the supplier compensate the distributor μv_0 increases with r_d when v_0 is fixed.

Next, we consider the impact of the parameter v_0. For the numerical example we consider, by virtue of (33) and (34), we can derive that, when $v_0 \in [-34.5, 56]$, both of the profits of the supplier and the distributor under the BBRS are higher than those under the decentralized case. The numerical results are presented in Figure 7. From the figure, we can see that the profit of the supplier decreases with v_0 and the profit of the distributor increases with v_0.

7.3 Bankruptcy Risks.

Because of the yield and demand uncertainties, the supplier and the distributor both face bankruptcy risks under the decentralized case and the BBRS. Since both of the supplier and the distributor are assumed to be risk neutral, they make decisions with the goal of maximizing expected profits, and their bankruptcy risks don't matter. However, as they use banking loans to finance their capital deficit, it is necessary to analyze the financing risks of suppliers and distributors from the bank's point of view. We use the bankruptcy probabilities to measure the bankruptcy risks of the supplier and the distributor. The following two propositions give their bankruptcy risks under the decentralized case and the BBRS contract.

Proposition 10 *Under the decentralized case, the bankruptcy probabilities of the supplier*

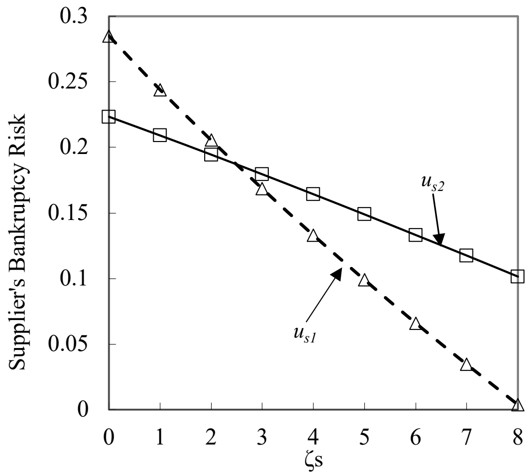

Figure 8: Bankruptcy risks vs. ζ_s

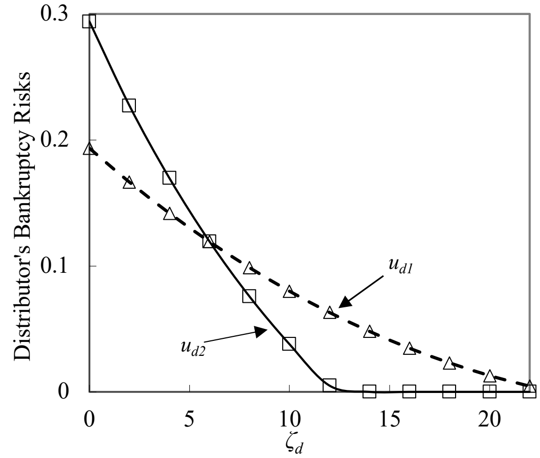

Figure 9: Bankruptcy risks vs. ζ_d

and the distributor are respectively

$$u_{s_1} = \begin{cases} F\left(\frac{(1+\tilde{r}_{p_1})(cq^*_{s_1}-\zeta_s)+c_r q_{d_1}}{(w+c_r)q^*_{s_1}}\right), & \text{if } \zeta_s < cq^*_{s_1}, \\ 0, & \text{if } \zeta_s \geq cq^*_{s_1}, \end{cases}$$

$$u_{d_1} = \begin{cases} \overline{F}\left(\frac{1}{k_1}\right) G\left(\frac{(1+\tilde{r}_{d_1})(wq_{d_1}-(1+r_p)\zeta_d)}{p}\right) & \\ \quad + \int_{\alpha_1}^{\frac{1}{k_1}} G\left(\frac{(1+\tilde{r}_{d_1})((w+c_r)q_{s_1}x - c_r q_{d_1} - (1+r_p)\zeta_d)}{p}\right) f(x)dx, & \text{if } (1+r_p)\zeta_d < wq^*_{d_1}, \\ 0, & \text{if } (1+r_p)\zeta_d \geq wq^*_{d_1}, \end{cases}$$

where $\alpha_1 = \frac{(1+r_p)\zeta_d - c_r q_{d_1}}{(w+c_r)q_{s_1}}$, *and* $q^*_{s_1}, q^*_{d_1}$ *are given by* (7), (13), *respectively.*

Proof of Proposition 10 is in Appendix A.6. In addition, we have the following result.

Proposition 11 *Under the BBRS, the bankruptcy probabilities of the supplier and the distributor are respectively*

$$u_{s_2} = \begin{cases} \int_0^{\alpha_2} f(x)\overline{G}(q^*_{s_2}x)dx, & \text{if } \zeta_s < cq^*_{s_2}, \\ 0, & \text{if } \zeta_s \geq cq^*_{s_2}, \end{cases}$$

$$u_{d_2} = \int_{\alpha_3}^{\infty} G\left(\frac{[(1+\tilde{r}_{d_2})w - \mu]q_{s_2}x + \mu v_0 - (1+\tilde{r}_{d_2})(1+r_p)\zeta_d}{p-\mu}\right) f(x)dx,$$

where

$$\alpha_2 = \frac{(1+\tilde{r}_{pd_2})(cq^*_{s_2} - \zeta_s) - \mu v_0}{(1+r_p)wq^*_{s_2}}, \quad \alpha_3 = \frac{(1+r_p)\zeta_d}{wq_{s_2}},$$

and $q^*_{s_2}$ *is given by* (27).

Proof of Proposition 11 is in Appendix A.7.

In Figure 8, we present the numerical results on the supplier's bankruptcy risks under both the decentralized case and the BBRS contract as the supplier's initial capital ζ_s

changes. Similarly, the corresponding numerical results for the distributor are presented in Figure 9. From the figures, we can see that for both the supplier and the distributor, the bankruptcy probabilities under the decentralized case and that under the BBRS are both decreasing with respect to their initial capital ζ_s and ζ_d, respectively. These are consistent with the results of Propositions 10 and 11, and are also consistent with intuitions.

Moreover, there are some interesting findings from Figures 8-9. From Figure 8, we can see that, the supplier's bankruptcy probability decreases faster under the decentralized case, if compared to the BBRS case. In addition, if the initial capital ζ_s is above a certain level (around 2.6), the supplier's bankruptcy probability under the BBRS case is actually higher than that under the decentralized case. It means that, when the supplier has a relatively higher level of initial capital, the BBRS may increase her bankruptcy probability. The reason is that, with higher initial capital, and knowing that all product are purchased under the BBRS, the supplier tends to be more aggressive and faces a higher risk.

However, it is a different story for the distributor. From Figure 9 we can see that, the distributor's bankruptcy risk decreases slower under the decentralized case, comparing to the BBRS case. In addition, if the initial capital ζ_d is above a certain level (around 6), the distributor's bankruptcy risk under the BBRS case is actually lower than that under the decentralized case, and it can achieve zero if the initial capital is large enough (around 14). It means that, when the distributor has a relatively higher level of initial capital, the BBRS can reduce her bankruptcy probability. The reason is that, with a relatively high initial capital, the distributor's benefit from the buyback of the unsold product excesses the cost of purchasing the over-produced product, so the distributor's bankruptcy risk is reduced. This is due to the special feature of risk-sharing of the proposed BBRS mechanism.

From the above discussion we can see that, the proposed BBRS mechanism can coordinate the supply chain with capital constraints, yield and demand uncertainties efficiently, and the total profit level can achieve that under the centralized case. By choosing an appropriate value of the parameter v_0, both the supplier and the distributor gain higher profits from the BBRS contract. However, in term of the bankruptcy risks, it seems like the BBRS would benefit more to the distributor than to the supplier.

Although it has been showed that the classical coordination contracts of revenue-sharing, buyback and quantity discount cannot coordinate the financially constrained supply chain in general (Kouvelis and Zhao, 2016; Xiao et al., 2017), we have showed that, the proposed BBRS coordination contract can efficiently coordinate the supply chain with yield uncertainty and financial constraints to achieve the profit under the centralized case. It is the risk-sharing feature that makes this coordination contract work. That is, the total risks from the yield uncertainty and the demand uncertainty are shared by both the supplier and the distributor. Moreover, the supplier and the distributor can negotiate to allocate the spillover profit of the supply chain under the BBRS so that both of them obtain higher profits than that under the decentralized case.

8 Conclusions.

A BBRS coordination contract is proposed in this paper, in order to improve a supply chain with yield and demand uncertainties, where both the supplier and the distributor face capital constraints. The optimal strategies and profits are derived and compared with the decentralized case and the centralized case. It has been showed that, the BBRS does improve the supply chain efficiently, and the total profit can achieve the level under the centralized case. The distributor and the supplier can negotiate to allocate the spillover profit between them so both of them are better off in terms of the profits. However, further analysis indicates that the BBRS more likely brings more benefit to the distributor than it does for the supplier in terms of the bankruptcy risks. In addition, the BBRS can increase or decrease the bankruptcy risks for both the supplier and the distributor, depending on their initial capital levels. Therefore, the BBRS coordination should be used with caution, if the bankruptcy risk is a concern.

There are some possible extensions that can be addressed in future research. For example, it is worth to investigate other coordination contracts for supply chains with capital constraints and yield uncertainty. In addition, as there are often some government stimulation policies such as agricultural subsidy, it would be meaningful to consider how to coordination the supply chain with capital constraints and yield uncertainty under some subsidy or support policies.

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Appendix.

A.1. Proof of Proposition 1.

From the definition of π_{s_1}, it is easy to see that

$$\frac{d\pi_{s_1}}{dq_{s_1}} = (1+r_d)[(w+c_r+c_h)\int_0^{\frac{q_{d_1}}{q_{s_1}}} xf(x)dx - (1+r_p)c - c_h],$$
$$\frac{d^2\pi_{s_1}}{dq_{s_1}^2} = -\frac{(1+r_d)(w+c_r+c_h)q_{d_1}^2 f(\frac{q_{d_1}}{q_{s_1}})}{q_{s_1}^3} < 0.$$

Thus, $\pi_{s_1}(q_{s_1})$ is a concave function of q_{s_1}. Set $\frac{d\pi_{s_1}}{dq_{s_1}} = 0$, we can get that $\int_0^{\frac{q_{d_1}}{q^*_{s_1}}} xf(x)dx = \frac{(1+r_p)c+c_h}{w+c_r+c_h}$. Denote $T(u) \equiv \int_0^u xf(x)dx$, then $T(u)$ is a monotone function of u. So (8) has a unique solution and the optimal $q^*_{s_1}$ is given by (7). □

A.2. Proof of Proposition 3.

By virtue of the constraint $q^*_{s_1} = k_1 q_{d_1}$, we can get that

$$\begin{aligned}\frac{d\pi_{d_1}}{dq_{d_1}} &= k_1 \int_0^{\frac{1}{k_1}} [(p+c_h)\overline{G}(k_1 q_{d_1} x) - (1+r_d)(w+c_r)]xf(x)dx \\ &\quad +\overline{F}\left(\frac{1}{k_1}\right)[(p+c_h)\overline{G}(q_{d_1}) - (1+r_d)(w+c_r) - c_h] + (1+r_d)c_r, \\ \frac{d^2\pi_{d_1}}{dq_{d_1}^2} &= -(p+c_h)\left[\overline{F}\left(\frac{1}{k_1}\right)g(q_{d_1}) + k_1^2\int_0^{\frac{1}{k_1}} g(k_1 q_{d_1} x)x^2 f(x)dx\right] < 0.\end{aligned}$$

Thus, under the decentralized case, the expected terminal cash flow of the distributor is a concave function of q_{d_1}. Set $\frac{d\pi_{d_1}}{dq_{d_1}} = 0$, we can get (13). □

A.3. Proof of Proposition 5.

From (16), we can get that

$$\frac{d\pi_{c_0}}{dq_{s_0}} = (p+c_h)\int_0^\infty xf(x)\overline{G}(q_{s_0}x)dx - (1+r_p)(1+r_d)c - c_h,$$
$$\frac{d^2\pi_{c_0}}{dq_{s_0}^2} = -(p+c_h)\int_0^\infty x^2 f(x)g(q_{s_0}x)dx < 0.$$

Thus, the expected terminal cash flow of the centralized supply chain is a concave function of q_{s_0}. Set $\frac{d\pi_{c_0}}{dq_{s_0}} = 0$, and we can get (17). □

A.4. Proof of Proposition 7.

From the definition of π_{s_2} and noting that $\lambda \leq w$, it is easy to see that

$$\begin{aligned}\frac{d\pi_{s_2}}{dq_{s_2}} &= (1+r_d)w - \mu\int_0^\infty xG(q_{s_2 x})f(x)dx - (1+r_p)(1+r_d)c, \\ \frac{d^2\pi_{s_2}}{dq_{s_2}^2} &= -\mu\int_0^\infty x^2 g(q_{s_2}x)f(x)dx < 0.\end{aligned}$$

Thus, π_{s_2} is a concave function of q_{s_2}. Set $\frac{d\pi_{s_2}}{dq_{s_2}} = 0$, and we can get (25). □

A.5. Proof of Proposition 9. Firstly, we prove that

$$\int_0^{\frac{1}{k_1}} [(p + c_h)\overline{G}(k_1 q_{d_1}^* x) - (1 + r_d)(w + c_r)]xf(x)dx > 0.$$

Now assume that $\int_0^{\frac{1}{k_1}} [(p + c_h)\overline{G}(k_1 q_{d_1}^* x) - (1 + r_d)(w + c_r)]xf(x)dx \leq 0$. Then from (13) we can get that $(p + c_h)\overline{G}(q_{d_1}^*) - (1 + r_d)(w + c_r) - c_h \geq 0$. Using the fact that $\overline{G}$ is non-increasing, we can get that

$$\begin{aligned}&\int_0^{\frac{1}{k_1}} [(p + c_h)\overline{G}(k q_{d_1}^* x) - (1 + r_d)(w + c_r)]xf(x)dx \\ &\quad > [(p + c_h)\overline{G}(q_{d_1}^*) - (1 + r_d)(w + c_r)] \int_0^{\frac{1}{k_1}} xf(x)dx \ > 0.\end{aligned}$$

It is a contradiction. So we must have

$$\int_0^{\frac{1}{k_1}} [(p + c_h)\overline{G}(k_1 q_{d_1}^* x) - (1 + r_d)(w + c_r)]xf(x)dx > 0.$$

Further, from (8) and $q_{s_1}^* = k_1 q_{d_1}^*$, we can get that

$$(p + c_h)\int_0^{\infty} \overline{G}(q_{s_1}^* x)xf(x)dx > (p + c_h)\int_0^{\frac{1}{k_1}} \overline{G}(q_{s_1}^* x)xf(x)dx > (1 + r_p)(1 + r_d)c.$$

In addition, from (27), we can get that

$$(p + c_h)\int_0^{\infty} \overline{G}(q_{s_2}^* x)xf(x)dx = (1 + r_p)(1 + r_d)c < (p + c_h)\int_0^{\infty} \overline{G}(q_{s_1}^* x)xf(x)dx. \quad (37)$$

If $q_{s_2}^* (= q_{s_0}^*) \leq q_{s_1}^*$, we must have

$$\overline{G}(q_{s_2}^* x) \geq \overline{G}(q_{s_1}^* x), \quad \forall x \in [0, \infty],$$

for that $\overline{G}$ is non-increasing. Then we can get that

$$(p + c_h)\int_0^{\infty} \overline{G}(q_{s_2}^* x)xf(x)dx \geq (p + c_h)\int_0^{\infty} \overline{G}(q_{s_1}^* x)xf(x)dx.$$

which is a contradiction with (37). Therefore, we can get that $q_{s_2}^* (= q_{s_0}^*) > q_{s_1}^*$. □

A.6. Proof of Proposition 10.

From (4), we can see that when $\zeta_s \geq c q_{s_1}^*$, the initial capital of the supplier is enough, so the supplier's bankruptcy probability is 0. On the other hand, when $\zeta_s < c q_{s_1}^*$, we can get that

$$u_{s_1} = Pr(w q_{s_1}^* X - c_r(q_{d_1} - q_{s_1} X) < (1 + \tilde{r}_{p_1})(c q_{s_1}^* - \zeta_s)) = F\left(\frac{(1 + \tilde{r}_{p_1})(c q_{s_1}^* - \zeta_s) + c_r q_{d_1}}{(w + c_r) q_{s_1}^*}\right).$$

From (10), we can see that when $(1+r_p)\zeta_d \geq wq^*_{d_1}$, $(1+r_p)\zeta_d \geq w\min(q^*_{d_1}, q^*_{s_1}X) - c_r(q_{d_1} - q_{s_1}X)^+$, the initial capital of the distributor is enough, so the distributor's bankruptcy probability is 0. When $(1+r_p)\zeta_d < wq^*_{d_1}$, we can get that

$$\begin{aligned} u_{d_1} &= Pr(pY < (1+\tilde{r}_{p_1})[w\min(q_{s_1}X, q_{d_1}) - c_r(q_{d_1} - q_{s_1}X)^+ - (1+r_p)\zeta_d]^+) \\ &= \overline{F}\left(\frac{1}{k_1}\right) G\left(\frac{(1+\tilde{r}_{d_1})(wq_{d_1} - (1+r_p)\zeta_d)}{p}\right) \\ &\quad + \int_{\frac{(1+r_p)\zeta_d - c_r q_{d_1}}{(w+c_r)q_{s_1}}}^{\frac{1}{k_1}} G\left(\frac{(1+\tilde{r}_{d_1})((w+c_r)q_{s_1}x - c_r q_{d_1} - (1+r_p)\zeta_d)}{p}\right) f(x)dx. \end{aligned}$$

A.7. Proof of Proposition 11.

From (23) and (26), when $\zeta_s < cq^*_{s_2}$, we can get that the supplier's bankruptcy risk as

$$\begin{aligned} u_{s_2} &= Pr((1+r_p)wq^*_{s_2}X - \mu(q^*_{s_2}X - Y)^+ + \mu v_0 < (1+\tilde{r}_{pd_2})\eta_{s_2}) \\ &= \int_0^{\alpha_2} f(x)\overline{G}(q_{s_2}x)dx. \end{aligned} \tag{38}$$

On the other hand, from (29), we can get that the bankruptcy risk of the distributor is

$$\begin{aligned} u_{d_2} &= Pr\Big(p\min(q_{s_2}X, Y) + \mu(q_{s_2}X - Y)^+ - \mu v_0 < (1+\tilde{r}_{d_2})\eta^-_{d_2}\Big) \\ &= \int_{\alpha_3}^{\infty} G\left(\frac{[(1+\tilde{r}_{d_2})w - \mu]q_{s_2}x + \mu v_0 - (1+\tilde{r}_{d_2})(1+r_p)\zeta_d}{p-\mu}\right) f(x)dx. \qquad \square \end{aligned}$$